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A SIMILARITY SOLUTION FOR A
SPHERICAL SHOCK WAVE

Richard Latter

P-633

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A SIMILARITY SOLUTION FOR A SPHERICAL SHOCK WAVE*

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ABSTRACT

The point-source, spherical shock wave moving into a constant density, γ -law gas is considered in the limit of infinite shock strength from the point of view of the Richtmyer-von Neumann viscosity technique. A similarity solution of this problem is shown to exist and is obtained for various boundary conditions with $\gamma = 1.4$. The solutions are obtained analytically in that part of the flow field not involving viscosity, and numerically in the other parts of the flow field. It is found that whereas all discontinuities of the physical parameters are removed by the viscosity there remain discontinuities in the slopes of these parameters at the shock-front. It is indicated, moreover, that the complete flow field depends upon the form and magnitude of the viscosity.

I. INTRODUCTION

The existence of shock waves in a hydrodynamic flow field introduces free-boundary discontinuities into the physical parameters of the system. Such discontinuities cause considerable analytic as well as numerical complications for the treatment of hydrodynamic problems. A means for avoiding these difficulties, particularly for numerical calculations, has been developed by Richtmyer and von Neumann⁽¹⁾. They observed that the addition of a

(1) R. D. Richtmyer and J. von Neumann, Journ. of Appl. Phys. 21 237 (1950).

particular viscosity-like term into the hydrodynamic equations could lead to

* This work was sponsored by the U. S. Air Force.

continuous shock-flow fields in which the discontinuities at the shock wave were removed and replaced by a region in which the physical parameters changed rapidly, but smoothly. Moreover, the change of the physical parameters through this transition region was, for plane flow fields, in agreement with the Rankine-Hugoniot relations.

Richtmyer and von Neumann obtained an analytic solution for flow field including the artificial viscosity only for a plane shock wave. In the present discussion, it is shown (using the viscosity technique) that in the limit of infinite shock strength, a point-source spherical shock flow has a similarity solution for a γ -law gas. The latter has been obtained by numerical and analytic integration of the ordinary differential equations resulting from the similarity of the flow.

An important consequence of the similarity solution described here is that it reveals that though the discontinuities of the physical parameters are removed from the flow field, the rates of change of these parameters have discontinuities. Moreover, the flow field itself is modified by the viscosity, approaching the Taylor⁽²⁾ strong-shock field only in the limit of vanishing

(2) G. I. Taylor, Proc. Roy. Soc. 201, 159 (1950).

viscosity.

II. FORMULATION

When the viscosity term suggested by Richtmyer and von Neumann is included into the hydrodynamic equations for a spherically symmetric flow in a γ -law medium, the hydrodynamic equations in Eulerian form become

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = - \frac{1}{\rho} \frac{\partial (p+q)}{\partial r}, \quad (1)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} = -\rho \left(\frac{\partial u}{\partial r} + \frac{2u}{r} \right), \quad (2)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} = \frac{\gamma p + (\gamma - 1)q}{\rho} \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right), \quad (3)$$

where p is the static pressure, ρ the density of the medium, u the material velocity, q the artificial viscosity, and (r, t) the space-time Eulerian coordinates. The caloric equation of state of the medium is assumed to be

$$e = p/(\gamma - 1)\rho, \quad (4)$$

where e is the internal energy per unit mass and $\gamma > 1$. The adiabatic equation of state is assumed to be

$$p/\rho^\gamma = \sigma(S), \quad (5)$$

where $\sigma(S)$ is a function of the entropy S only. The quantity q in Eqs. (1) - (3) is the viscosity term. That dependence of q on p, ρ , and u which prohibits discontinuities in the physical parameters is not uniquely prescribed by Richtmyer and von Neumann. The form for q adopted for the present discussion is consistent with their requirements and is

$$q = \frac{1}{2} K^2 \rho r^2 \frac{\partial u}{\partial r} \left(\left| \frac{\partial u}{\partial r} \right| - \frac{\partial u}{\partial r} \right). \quad (6)$$

The arguments presented here are not restricted to the form of q in Eq. (6), but can be generalized to include other possibilities. However, some of the detailed results depend upon the specific form of q which is used; in particular, not all forms for q admit of similarity solutions.

The possibility of a similarity solution of Eqs. (1) - (3) and (6) is suggested by the calculations of Taylor⁽²⁾ on the point-source, strong-shock

problem, in which the shock is treated as a discontinuity. It may be observed from Eq. (6) that the Taylor equations and the present ones are identical in the region between the origin and the Taylor shock-front since $q = 0$ in this region, as follows from the property $\frac{\partial u}{\partial x} > 0$ for the material velocity behind the Taylor strong shock. This observation indicates that the Taylor similarity variables may be suitable also for the present problem. Thus the Eqs. (1) - (3) and (6) are examined for a similarity solution of the form

$$\frac{\rho(r,t)}{\rho_0} = \frac{f(x)}{R^\beta}, \quad (7)$$

$$\frac{p(r,t)}{\rho_0} = \psi(x), \quad (8)$$

$$u(r,t) = \frac{\phi(x)}{R^\alpha}, \quad (9)$$

where $x = r/R$, R is a function of time only, which at present need have no relation to the shock position, ρ_0 is an arbitrarily prescribed constant with the dimensions of density, and α and β are constants yet to be determined. To simplify the subsequent calculations the result $\beta = 2\alpha$, which may be easily derived from the arguments to follow, is assumed without proof. The expressions in Eqs. (7) - (9) are now substituted into Eqs. (1) - (3) and (6), using the relationships, which result from the change of independent variables from (r,t) to (x,t) ,

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \frac{xR}{R} \frac{\partial}{\partial x},$$

$$\frac{\partial}{\partial r} = \frac{1}{R} \frac{\partial}{\partial x},$$

where $\dot{R} = dR/dt$. Equation (1) then becomes in terms of the new variables

$$-\alpha \dot{R} R^\alpha - x \phi' R^\alpha \dot{R} + \phi \phi' + \frac{f' + g'}{\psi} = 0, \quad (10)$$

where the prime indicates differentiation with respect to x . The quantity g is related to q by the equation

$$\frac{q}{\rho_0} = \frac{K^2}{2R^{2\alpha}} + x^2 \phi' \left(|\phi'| - \phi' \right) = \frac{g(x)}{R^{2\alpha}}, \quad (11)$$

which arises directly from Eq. (6) using Eqs. (7) - (9). If, now, it is assumed that

$$R^\alpha \dot{R} = A, \text{ a constant}, \quad (12)$$

then Eq. (10) becomes an ordinary differential equation in the independent variable x , namely,

$$-A(\alpha\phi + x\phi') + \phi\phi' + \frac{f' + g'}{\psi} = 0. \quad (13)$$

Quite similarly, Eqs. (2) and (3) become, respectively,

$$-A x \psi' + \phi \psi' + \psi \phi' + 2 \frac{\phi \psi}{x} = 0 \quad (14)$$

and

$$-A 2\alpha f + (\phi - Ax)f' - \frac{7f + (7-1)g}{\psi} (\phi - Ax)\psi' = 0. \quad (15)$$

The constant α is determined by requiring a constant energy in the hydrodynamic flow. If E is the total energy, then

$$E = 4\pi \int_0^\infty r^2 dr (\rho u^2/2 + p/(\gamma-1)),$$

which becomes in terms of the present similarity variables

$$E = 4\pi \frac{R^3}{R^{2\alpha}} \rho_0 \int_0^\infty x^2 dx (\psi \phi'^2/2 + f/(\gamma-1)). \quad (16)$$

Since R is a function of time, this equation requires that $\alpha = 3/2$.

If new dimensionless quantities

$$F = \frac{f}{A^2}, \quad G = \frac{g}{A^2}, \quad \text{and } \phi = \frac{\phi}{A}$$

are introduced into Eqs. (11) and (13) - (16), these equations read

$$G = \frac{K^2}{2} x^2 \left(\phi' - \phi \right), \quad (11')$$

$$-\left(\frac{3}{2} \phi + x \phi' \right) \phi' + \frac{F' + G'}{x} = 0, \quad (13')$$

$$(\phi - x) \phi' + \phi' + 2 \frac{\phi}{x} = 0, \quad (14')$$

$$-3F + (\phi - x)F' - \frac{\gamma F + (\gamma - 1)G}{x} (\phi - x) \phi' = 0, \quad (15')$$

and

$$K = 4\pi \rho_0 A^2 \int_0^\infty x^2 dx \left(\phi'^2/2 + F/(\gamma - 1) \right). \quad (16')$$

The existence of a solution of Equations (13') - (15') for a diverging flow from a point source with a shock-discontinuity at $x = 1$ has been demonstrated by Taylor. But for such a diverging flow $\frac{\partial \phi}{\partial x} > 0$, so that $G = 0$, and therefore the Taylor solution constitutes also a solution of the present problem in the region from $x = 0$ to $x = 1$, where R is identified with the position of the Taylor shock-front. In the Taylor solution the quantities F , ϕ and ψ at $x = 1$ are consistent with the Rankine-Hugoniot conditions for a strong shock. The presence of viscosity removes the discontinuities in the physical quantities and consequently, the Rankine-Hugoniot conditions have no special significance in the viscosity formalism. However, if the viscosity is to leave the flow field unaltered, then the Rankine-Hugoniot conditions should apply across the shock-transition region wherein the viscosity is effective. In a similarity

problem of the present type it is clear that this situation is not possible since the width of the transition region will be a constant with respect to the similarity variable $x = \frac{r}{R}$. The transition region will therefore contain an amount of mass given by

$$m(r,t) = 4\pi \int r^2 dr \rho(r,t) ,$$

which in terms of the similarity variables is

$$m(r,t) = 4\pi \rho_0 R^3 \int x^2 dx \psi(x) .$$

The limits of integration are fixed by the width of the transition region and depend, consequently, only upon x . The time dependence of R given in Eq. (12) implies then that the mass, $m(r,t)$, within the transition region varies with time. For the validity of the Rankine-Hugoniot conditions across the transition region, $m(r,t)$ would have to be a constant, but this condition has been proved to be violated. This impossibility of meeting the Rankine-Hugoniot conditions, is, of course, a property of the assumed type of similarity expressed by Eqs. (7) - (9) as well as of the specific form for q of Eq. (6). The restrictions on the form of q leading to the Rankine-Hugoniot conditions are still not known in general and certainly need further exploration.

Before studying the behavior of the differential equations, it is convenient to formulate the boundary conditions which these equations must satisfy. Namely, all physical quantities must be bounded and continuous. While the artificial viscosity is not a physical quantity, the spirit of the Richtmyer-von Neumann formalism requires treating it as such and therefore q is also assumed continuous. For the presently considered diverging flow-field, the physical quantities must satisfy the additional conditions that in the neighborhood of $x = 0$, $\psi = 0$, $F > 0$, $G = 0$, and $\epsilon > 0$, and in front of the

flow the medium is undisturbed so that $\phi = 0$, $F \geq 0$, $G = 0$, and ψ is a constant. From Eq. (7) it is readily observed that the similarity of the flow requires $F = 0$ for large x in the undisturbed medium.

The treatment of Equations (11') and (13') - (15') will be divided into the consideration separately of the regions wherein the viscosity is absent and wherein it is present. The region without viscosity will be treated first and will be solved analytically. The region with viscosity will then be explored and joined on by numerical integration to the rest of the flow-field.

III. THE FLOW-FIELD WITHOUT VISCOSITY

From the boundary conditions for the diverging flow-field, it follows that in a neighborhood of $x = 0$, the gradient of the velocity is positive and therefore the viscosity term G is zero. The equations defining the flow then simplify to

$$-\left(\frac{3}{2}\phi + x\phi'\right) + \phi\phi' + \frac{F'}{\psi} = 0 \quad , \quad (17)$$

$$(\phi - x)\psi' + \psi\phi' + 2\frac{\phi}{x} = 0 \quad , \quad (18)$$

$$-3F + (\phi - x)F' - \frac{\gamma F}{\psi}(\phi - x)\psi' = 0 \quad . \quad (19)$$

These equations admit of a straightforward analytic integration. Thus by dividing Eq. (18) by $\psi(\phi - x)$ and by adding the result to Eq. (19), after dividing the latter by $F(\phi - x)$,

$$\frac{F'}{F} - (\gamma - 1)\frac{\psi'}{\psi} + \frac{(x - \phi)'}{x - \phi} + \frac{2}{x} = 0 \quad .$$

This equation integrates directly to

$$x^2(x - \phi) \frac{F}{\psi^{\gamma-1}} = a \quad . \quad (20)$$

A second integral of Eqs. (17) - (19) may be obtained by multiplying Eq. (17)

with ψ and by adding to Eq. (19) multiplied by $\frac{1}{\gamma-1}$. Simplifying the result with the aid of Eq. (18) gives

$$\frac{d}{dx} x^2 \left\{ F \phi - (x-\phi) \left(\frac{F}{\gamma-1} + \frac{1}{2} \psi \phi^2 \right) \right\} = 0.$$

Integration of this equation, using the condition $\phi = 0$ at $x = 0$ yields

$$F \phi - (x - \phi) \left(\frac{F}{\gamma-1} + \frac{1}{2} \psi \phi^2 \right) = 0. \quad (21)$$

Alternatively, this equation is an immediate consequence of the conservation of energy if this conservation is expressed in integral form. Finally the third integral is obtained by direct substitution of Eqs. (20) and (21) into Eqs. (17) - (19). Thus, first eliminate F' and ψ' from Eq. (17) using Eqs. (18) and (19); then

$$\psi = \frac{\frac{3}{2} \phi (x-\phi) \frac{\psi}{F} + (3-2\gamma) \frac{\phi}{x}}{\gamma - (x-\phi)^2 \frac{\psi}{F}}. \quad (22)$$

Expressing ψ and F in terms of ϕ and x gives for ϕ

$$\phi = -\frac{\phi}{x} \frac{2\gamma(\gamma-1) \left(\frac{\phi^2}{x} \right) - 3(2\gamma-1) \frac{\phi}{x} + 3}{\gamma(\gamma+1) \left(\frac{\phi^2}{x} \right) - 2(\gamma+1) \frac{\phi}{x} + 2}. \quad (23)$$

This is a standard homogeneous equation and readily integrated giving

$$x = b \frac{\frac{\phi}{x} - \frac{1}{\gamma}}{(13\gamma^2 - 7\gamma + 12)/5(2\gamma+1)(3\gamma-1)} \quad (24)$$

$$\frac{\phi}{x} = \frac{2/5}{\frac{\phi}{x} - \frac{5}{3\gamma-1}}$$

Eqs. (20), (21) and (24) constitute a complete analytic solution of the differential equations, Eqs. (17) - (19). Since F and ψ are intrinsically positive, the constant a of Eq. (20) is positive.

The behavior of the flow-field in the present region is defined by fixing the two arbitrary integration constants a and b . These are in turn specified by requiring definite values for ϕ and F at a prescribed position x . It is clear therefore that the Eqs. (17) - (19) describe a two parameter family of flow-fields. No further restriction of these parameters is possible before discussing the region containing viscosity.

Only the properties of the present solution relevant to the viscosity formalism will be considered. First, since $\phi = 0$ at $x = 0$, Eq. (24) requires that $\frac{\phi}{x} = \frac{1}{\gamma}$ at $x = 0$. Solving Eq. (21) for $\frac{F}{\psi}$ gives

$$\frac{F}{\psi} = \frac{\gamma-1}{27} x^2 \frac{\left(\frac{\phi}{x}\right)^2 (1 - \frac{\phi}{x})}{\frac{\phi}{x} - \frac{1}{\gamma}}$$

and, moreover, from the positivity of F and ψ , this equation shows that

$$\frac{1}{\gamma} \leq \frac{\phi}{x} \leq 1 \quad (25)$$

Combining Eqs. (24) and (25) leads to the restriction

$$\frac{1}{\gamma} \leq \frac{\phi}{x} \leq \text{Min}(1, \frac{5}{2\gamma-1}) \quad (26)$$

Using this equation with Eq. (23) it is shown immediately that

$$\phi' > 0, \quad (27)$$

for all $x \geq 0$ provided that the physical requirement $\gamma > 1$ is met. This result is established by observing first that at $x = 0$, $\frac{\phi}{x} = \frac{1}{\gamma}$ and $\phi' = \frac{1}{\gamma} > 0$.

If then ϕ' is to violate Eq. (27), there must exist a point at which $\phi' = 0$.

But this is possible only if

$$27(\gamma-1) \left(\frac{\phi}{x}\right)^2 - 3(2\gamma-1) \frac{\phi}{x} + 3 = 0$$

Solving this equation for $\frac{\phi}{x}$ yields

$$\frac{\phi}{x} = \frac{3(2\gamma-1) \pm \sqrt{3(4\gamma^2-4\gamma+3)}}{4\gamma(\gamma-1)}.$$

It may be demonstrated analytically from this expression or simply by plotting the two roots as a function of γ that for $\gamma > 0$

$$\left(\frac{\phi}{x}\right)_+ > \frac{5}{3\gamma-1} \quad \text{and} \quad \left(\frac{\phi}{x}\right)_- < \frac{1}{\gamma}.$$

But this result implies that for no value of $\frac{\phi}{x}$ in the allowed range expressed by Eq. (26) does $\phi' = 0$.

Since the viscosity term of Eq. (11') remains zero so long as $\phi' > 0$, there is no way for the viscosity to enter except by the introduction of discontinuities in the slopes of the physical quantities. This indicates a deficiency of the viscosity formalism which in numerical applications assumes continuity not only of the physical quantities themselves but also of their slopes.

IV. THE FLOW-FIELD WITH VISCOSITY

From the above arguments it has been established that Eqs. (13') - (15') are solved by a Taylor-type solution in the region $x \geq 0$ wherein $\phi = 0$. But in order to introduce the viscosity it has been proved necessary to admit discontinuous slopes. The position at which the discontinuities are located may be prescribed arbitrarily as $x = 1$ since this affects only the magnitude of R which is at present arbitrary and fixed only after specifying the constant A . The problem remaining is then the solution for the region $x \geq 1$ in which the viscosity is effective. This region comprises the transition from the continuous flow-field to the undisturbed medium in front of the flow. For this region $\phi' \geq 0$ and therefore Eq. (11') may be written

$$\dot{\phi} = -\frac{1}{Kx} \sqrt{\frac{G}{\psi}} \quad (11'')$$

At the connection point, $x = 1$, $G = 0$, as required by continuity of the physical quantities and therefore $\dot{\phi} = 0$ at this point. This in turn determines the magnitude of the jump in the slope $\dot{\phi}$, which combined with the differential equations prescribes the jumps of the slopes of F , ψ and G .

The solution of Eqs. (11'') and (13') - (15') for the transition region must meet the boundary conditions that for sufficiently large x , the medium is undisturbed so that $\phi = 0$, $F \geq 0$, $G = 0$ and ψ is a constant, and for $x = 1$ the solution agrees with the solution for the region $x = 0$ to $x = 1$, wherein $G = 0$. It may be shown quite generally by a series expansion for large x that all solutions of Eqs. (11'') and (13') - (15') which extend to infinity behave asymptotically as

$$\phi \sim \frac{\phi_0}{x^4}, \quad F \sim \frac{f_0}{x^3}, \quad G \sim \frac{g_0}{x^8}, \quad \text{and } \psi \sim \psi_0 + \frac{\psi_1}{x^5}, \quad (28)$$

where ϕ_0 , f_0 , g_0 , ψ_0 , and ψ_1 are constants. This result may be verified by direct substitution into the basic equations. This, of course, establishes that the medium in front of the shock-transition region is undisturbed and, moreover, is at zero pressure, which implies zero temperature. However, not every solution extends to infinity nor is physically admissible since ϕ , F , G and ψ must be everywhere positive in order to correspond to a diverging flow-field with positive pressures and densities. It may be shown that the physically admissible solutions would extend to infinite x . For admissible solutions, the values of the quantities ϕ , F , G and ψ at $x = 1$ must be restricted. To examine this restriction, it is observed first that certain restrictions on these quantities already exist. Namely, $G = 0$ at $x = 1$. Also

$$(F + G) \dot{\phi} - (x - \phi) \left(\frac{F}{\gamma - 1} + \frac{1}{2} \psi \dot{\phi}^2 \right) = 0, \quad (29)$$

or at $x = 1$

$$F \phi - (1 - \phi) \left(\frac{F}{\psi - 1} + \frac{1}{2} \psi^2 \right) = 0 \quad (30)$$

Eq. (29) follows from Eqs. (13') - (15') and Eq. (28) by a derivation completely analogous to that leading to Eq. (21); and Eq. (30) is a consequence of $G = 0$ at $x = 1$. Finally, it is observed from Eqs. (11'') and (13') - (15') that all solutions with the same values of F/ψ , G/ψ and ϕ at $x = 1$ lead to the same functions $\frac{F(x)}{\psi(1)}$, $\frac{G(x)}{\psi(1)}$, $\frac{\psi(x)}{\psi(1)}$ and $\phi(x)$. Since $G = 0$ at $x = 1$ and Eq. (30) relates F/ψ to ϕ at $x = 1$, the only independent degree of freedom remaining for the choice of the quantities at $x = 1$ with which to obtain an admissible solution is the specification of ϕ at $x = 1$. That, in fact, a choice of ϕ can be made which leads to an admissible solution has not been established analytically. However, a convincing demonstration of the existence of such a choice is provided by the explicit numerical solutions discussed in the next section. It may be considered proved, therefore, that Eqs. (11'') and (13') - (15') admit of solutions which consist of a continuous hydrodynamic flow from $x = 0$ to $x = 1$, and which exhibit a continuous transition region (except for slope discontinuities) from the flow conditions at $x = 1$ to an undisturbed medium at zero pressure for large x .

V. RESULTS AND DISCUSSION

To specify the diverging flow field the constants a and b of Eqs. (20) and (24) (which are equivalent to $\phi(1)$ and $F(1)/\psi(1)$) and the constant A of Eq. (12) must be fixed. The energy K of Eq. (16') fixes one relationship amongst these constants and the specification of the density of the undisturbed medium fixes another relationship. Finally, the requirement of a physically admissible solution in the transition region fixes the last constant. In the limit of vanishing viscosity, $K = 0$, the solutions, of course, become identical with the familiar strong-shock flow fields with a jump discontinuity

at the shock front, $x = 1$.

To illustrate the behavior of the present similarity problem, the solutions of Eqs. (11') and (13') - (15') with $\gamma = 1.4$ were obtained from Eqs. (20), (21) and (24) in the region without viscosity and by numerical integration using the Runge-Kutta method in the viscosity region. For $K = 0$, that is, no viscosity, the boundary conditions $\phi(1) = \frac{2}{\gamma+1}$, $F(1) = \frac{2}{\gamma+1}$, and $v(1) = \frac{\gamma+1}{\gamma-1}$ define the solution which is shown in Fig. 1. This is just the solution obtained by Taylor. In addition three non-zero values of K were used, namely, $K = 0.00349$, 0.0349 , and 0.349 . Figures 2 - 4 show the solutions for the diverging flow obtained from Eqs. (20), (21) and (24) in the region $0 \leq x \leq 1$ and obtained by numerical integration of Eqs. (11'') and (13') - (15') in the region $x \geq 1$. In tables I - IV, the solutions for these four values of K are tabulated. The values of the integral of Eq. (16') for these cases are

$$\begin{aligned}
 \frac{E}{4K\rho_0 A^2} &= 0.423 & , & \text{ for } K = 0.00000 & , \\
 &= 0.446 & , & \text{ for } K = 0.00349 & , \\
 &= 0.642 & , & \text{ for } K = 0.0349 & , \\
 &= 1.650 & , & \text{ for } K = 0.349 & .
 \end{aligned}$$

A comparison of these solutions with the Taylor solution, provides an indication of the influence of the viscosity on the flow-field. In particular, for the Taylor problem, ρ_0 is the density of the undisturbed medium and $v(1)\rho_0$ is the density of the shocked medium. These densities should be compared with $v(\infty)\rho_0$ and $v(1)\rho_0$ of the present problem, which are the densities of the undisturbed and shocked media, respectively. The ratio of these densities is 6 for the Taylor problem, but for $K = 0.00349$ it is 5.693, for $K = 0.0349$ it is

4.054, and for $K = 0.349$ it is 1.137. The use of the viscosity formalism is seen from these ratios, as well as from the complete solutions in Tables I - IV, to introduce considerable modification of the flow-field for large K , but negligible changes for small K .

The important consequence of the present discussion is the quantitative estimate of the influence of the viscosity on the flow field. Perhaps as significant is the proof that the viscosity technique may possess the deficiency in spherical flow fields that the physical parameters have discontinuous slopes at the shock-transition region, even though these parameters are continuous. Thus the usual numerical integration of Eqs. (1) - (3) and (6), carried out by replacing these equations with difference equations, cannot lead to a solution of the differential equations if the slope discontinuities are disregarded. It would appear therefore that these discontinuities should lead to inaccuracies in numerical applications of the viscosity technique, which ignores them. However, from H. Brode's⁽³⁾ results on the numerical

(3) H. Brode, to be published.

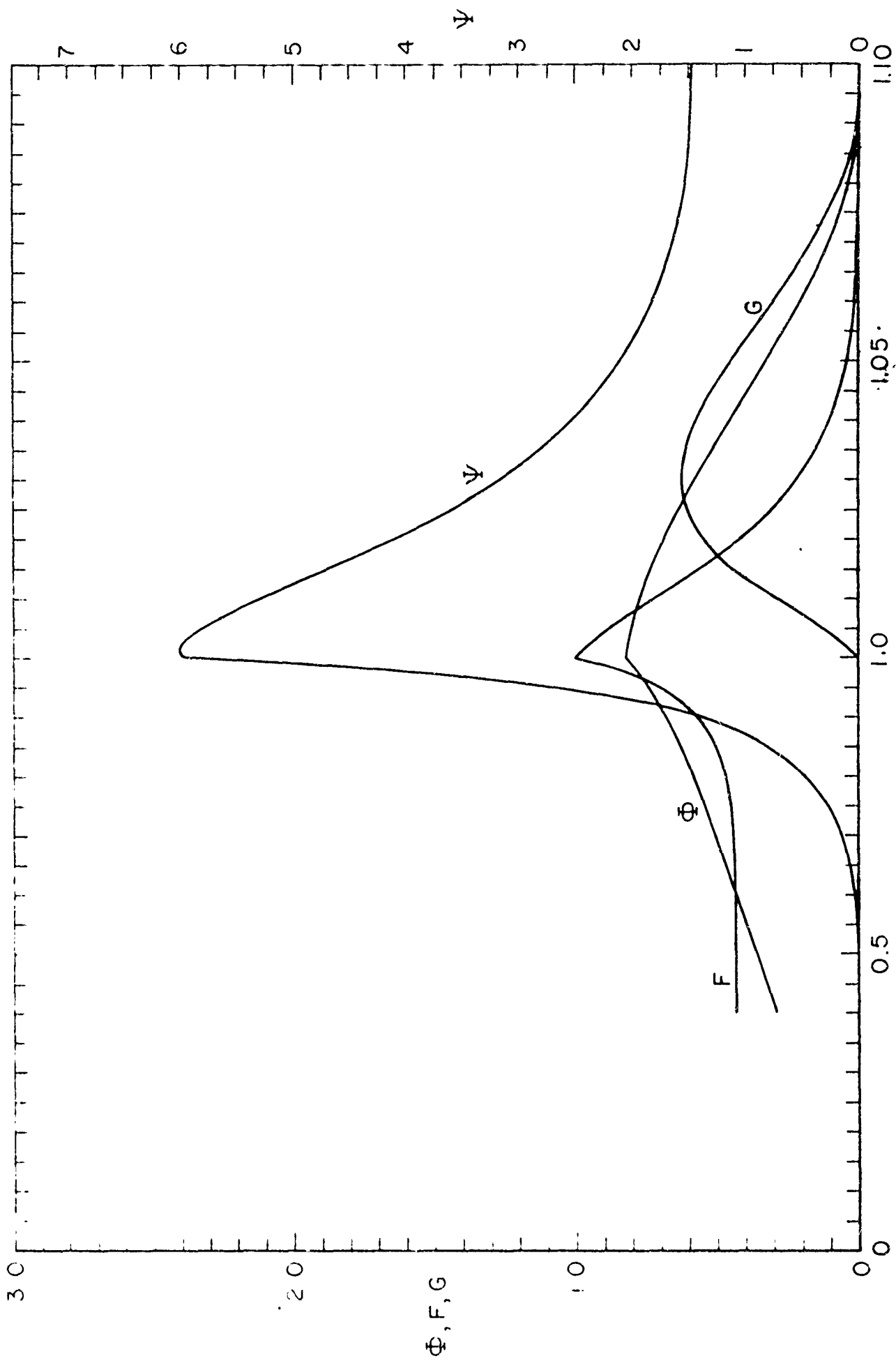
integration of the hydrodynamic Eqs. (1) - (3) using the viscosity technique for a spherical explosion, it appears that any errors from this source are probably unimportant. This conclusion should be qualified somewhat since in Brode's integration, the radius to the transition region was large and the neighborhood of the shock-front was approximately planar so that any discontinuities would be small. Moreover, the form of viscosity used by Brode differed from the present one, although the form which he used also suffered from not leading to a constant mass, within the transition region.

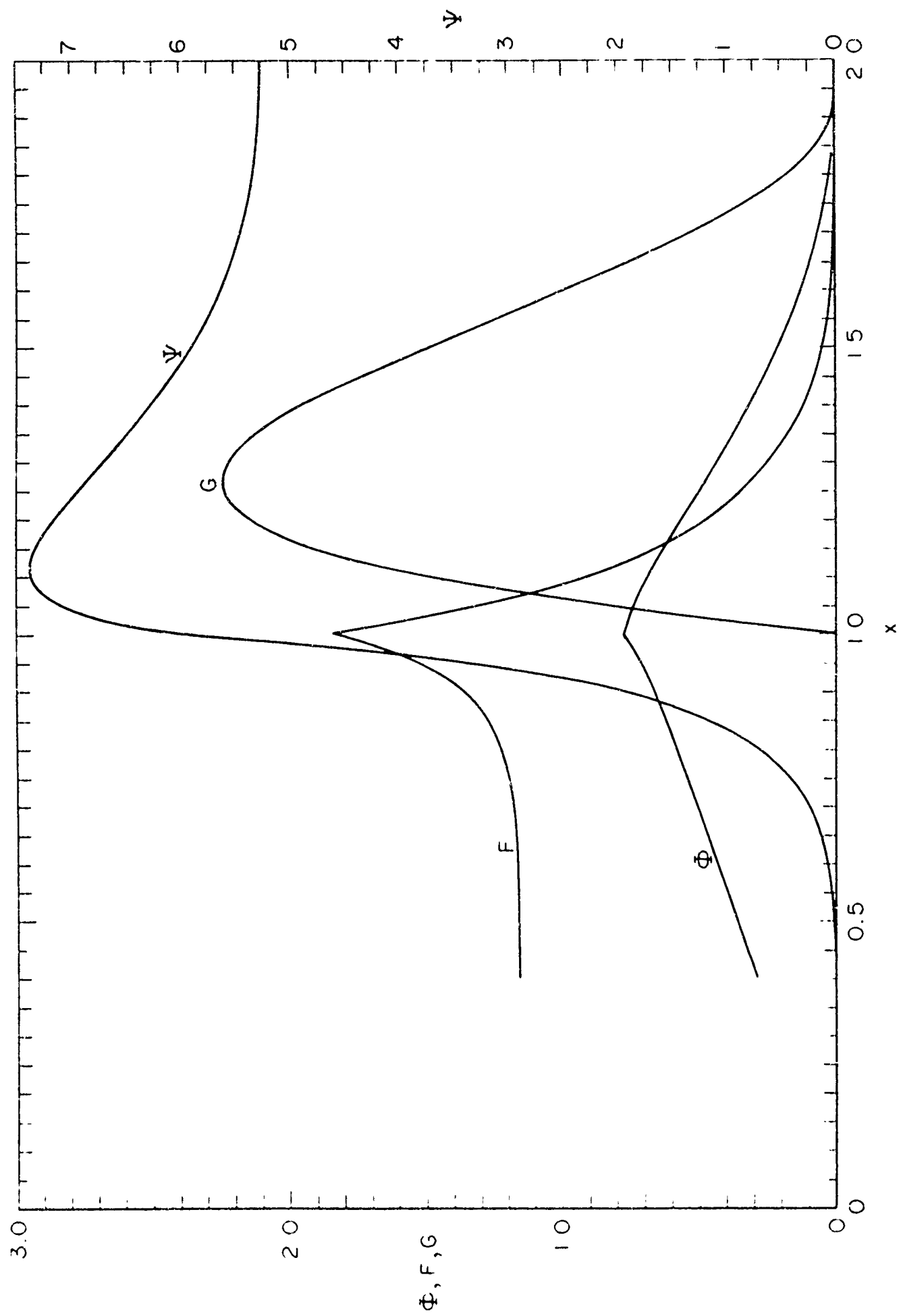
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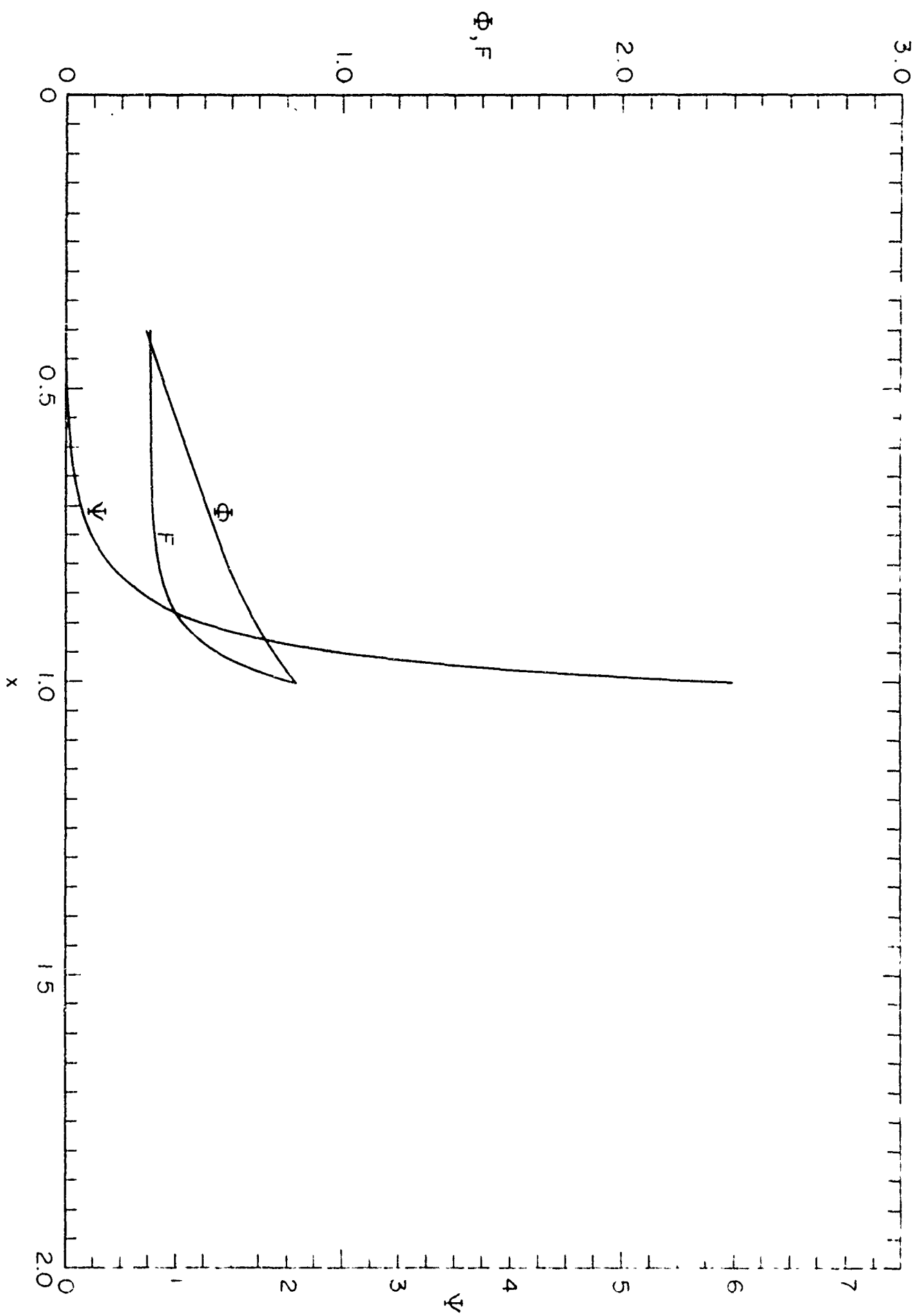
The author wishes to express his appreciation to Mr. Jack Little and Miss Ruth Merrill for numerical assistance.

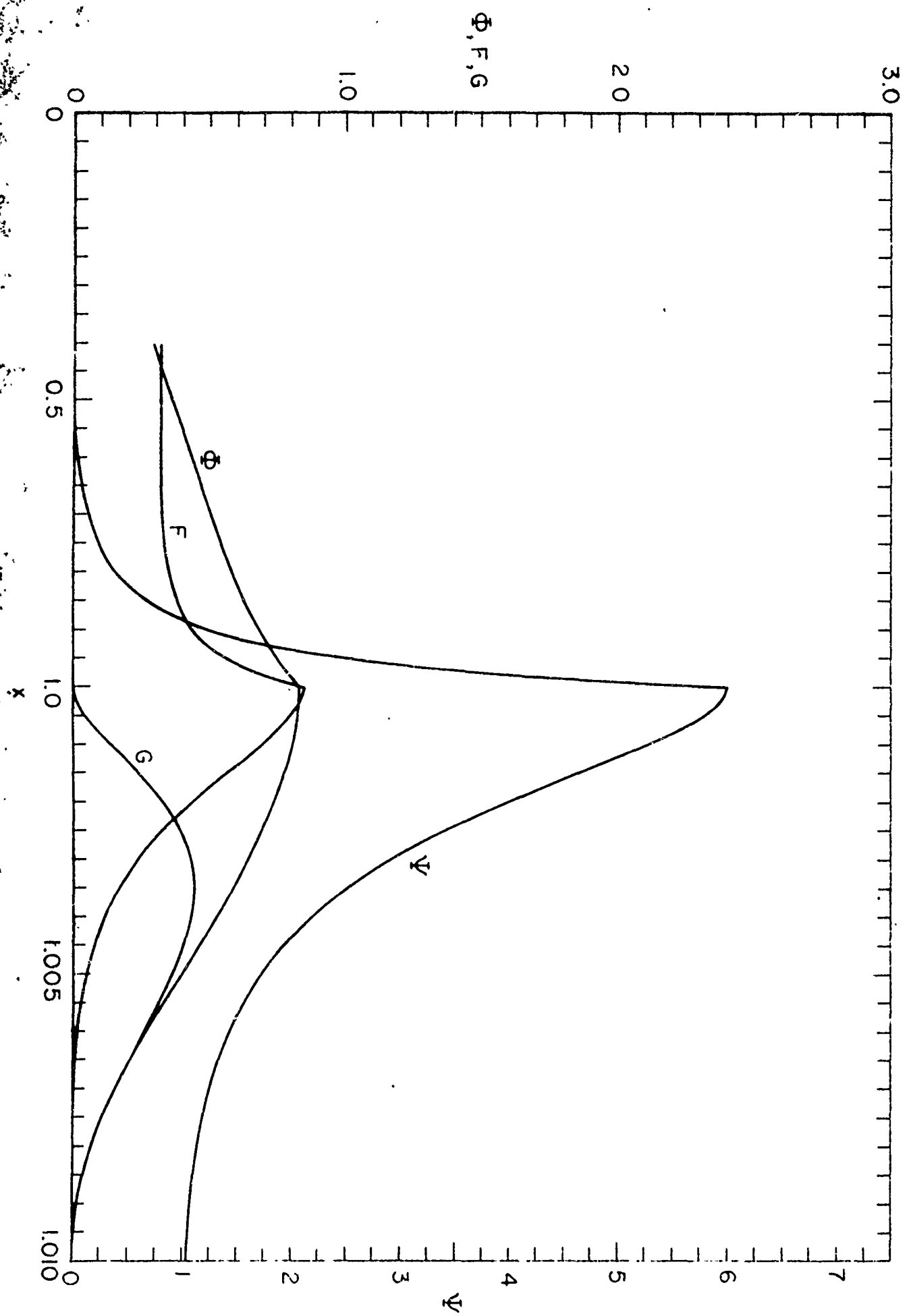
Captions for Figures

- Figure 1 - Solution for $K = 0$
- Figure 2 - Solution for $K = 0.00349$
- Figure 3 - Solution for $K = 0.0349$
- Figure 4 - Solution for $K = 0.349$









FRAMES

Table I : Solution for $K = 0$

	x	ξ	Ψ	F	G	
	.400	.287	.002	.305		
	.420	.301	.003	.305		
	.440	.316	.004	.305		
	.460	.330	.005	.305		
	.480	.344	.008	.305		
	.500	.358	.010	.305		
	.520	.372	.014	.305		
	.540	.387	.018	.306		
	.560	.401	.024	.306		
	.580	.415	.031	.306		
	.600	.430	.041	.307		
	.620	.444	.052	.308		
	.640	.459	.066	.308		
	.660	.474	.084	.310		
	.680	.488	.106	.311		
	.700	.503	.132	.313		
	.720	.519	.165	.316		
	.740	.534	.206	.320		
	.760	.550	.255	.324		
	.780	.567	.316	.330		
	.800	.584	.392	.337		
	.820	.602	.488	.347		
	.840	.622	.608	.359		
	.860	.642	.754	.374		
	.880	.665	1.032	.423		
	.900	.710	1.600	.460		
	.920	.737	2.140	.510		
	.940	.763	2.649	.543		
	.960	.788	3.139	.536		
	.980	.814	3.511	.550		
	.990	.830	3.706	.565		
	.995	.835	3.885	.581		
	.998	.840	4.080	.598		
	.999	.842	4.294	.617		
	.999	.843	4.527	.637		
	.999	.844	4.784	.659		
	.999	.845	5.066	.682		
	.999	.846	5.377	.708		
	.999	.847	5.721	.735		
	.999	.848	6.103	.765		
	.999	.849	6.527	.798		
	1.000	.850	6.990	.833	0.000	

Table II : Solution for $K = 0.0019$

X	$\frac{1}{X}$	Y	Z	G
.100	.487	.001	.318	
.110	.451	.003	.318	
.120	.316	.004	.318	
.130	.330	.006	.318	
.140	.344	.008	.319	
.150	.358	.010	.319	
.160	.372	.011	.319	
.170	.387	.019	.319	
.180	.401	.025	.319	
.190	.415	.032	.320	
.200	.430	.042	.320	
.210	.444	.053	.321	
.220	.459	.062	.322	
.230	.474	.086	.323	
.240	.488	.108	.325	
.250	.503	.136	.327	
.260	.519	.169	.330	
.270	.534	.210	.333	
.280	.550	.261	.338	
.290	.567	.324	.344	
.300	.584	.401	.353	
.310	.602	.499	.361	
.320	.620	.621	.374	
.330	.640	.778	.390	
.340	.661	.982	.411	
.350	.684	1.254	.439	
.360	.709	1.625	.476	
.370	.736	2.118	.527	
.380	.764	2.878	.580	
.390	.791	2.418	.553	
.400	.817	2.570	.567	
.410	.843	2.734	.582	
.420	.865	2.913	.599	
.430	.887	3.108	.616	
.440	.908	3.321	.635	
.450	.929	3.554	.655	
.460	.949	3.809	.677	
.470	.969	4.082	.702	
.480	.988	4.377	.728	
.490	.810	4.700	.754	
.500	.817	5.115	.784	
.510	.824	5.534	.816	
.520	.832	6.000	.852	
.530	.840	6.866	.899	
.540	.848	5.509	.744	
.550	.856	5.029	.616	
.560	.864	4.503	.511	
.570	.872	3.988	.441	
.580	.880	3.518	.352	
.590	.888	3.105	.276	
.600	.896	2.751	.213	
.610	.904	2.451	.166	
.620	.912	2.199	.121	
.630	.920	1.988	.089	
.640	.928	1.811	.064	
.650	.936	1.663	.045	
.660	.944	1.539	.031	
.670	.952	1.436	.021	
.680	.960	1.349	.013	
.690	.968	1.277	.008	
.700	.976	1.218	.004	
.710	.984	1.169	.002	
.720	.992	1.130	.001	
.730	.999	1.100	.000	
.740	.001	1.078	.000	
.750	.010	1.063	.000	
.760	.002	1.055	.000	
.770	.001	1.054	.000	
.780				
.790				
.800				
.810				
.820				
.830				
.840				
.850				
.860				
.870				
.880				
.890				
.900				
.910				
.920				
.930				
.940				
.950				
.960				
.970				
.980				
.990				
1.000				0.000
1.0001				.031
1.0002				.097
1.0003				.179
1.0004				.260
1.0005				.331
1.0006				.385
1.0007				.441
1.0008				.499
1.0009				.561
1.0010				.616
1.0011				.677
1.0012				.744
1.0013				.816
1.0014				.899
1.0015				.984
1.0016				1.078
1.0017				1.181
1.0018				1.299
1.0019				1.436
1.0020				1.599
1.0021				1.788
1.0022				2.011
1.0023				2.277
1.0024				2.588
1.0025				2.944
1.0026				3.344
1.0027				3.799
1.0028				4.309
1.0029				4.874
1.0030				5.494
1.0031				6.169
1.0032				6.900
1.0033				7.688
1.0034				8.534
1.0035				9.444
1.0036				10.419
1.0037				11.459
1.0038				12.564
1.0039				13.734
1.0040				14.969
1.0041				16.269
1.0042				17.634
1.0043				19.064
1.0044				20.559
1.0045				22.119
1.0046				23.744
1.0047				25.434
1.0048				27.189
1.0049				29.009
1.0050				30.894
1.0051				32.844
1.0052				34.859
1.0053				36.939
1.0054				39.084
1.0055				41.294
1.0056				43.569
1.0057				45.909
1.0058				48.314
1.0059				50.784
1.0060				53.319
1.0061				55.919
1.0062				58.584
1.0063				61.314
1.0064				64.109
1.0065				66.969
1.0066				69.894
1.0067				72.884
1.0068				75.939
1.0069				79.059
1.0070				82.244
1.0071				85.494
1.0072				88.809
1.0073				92.189
1.0074				95.634
1.0075				99.144
1.0076				102.719
1.0077				106.359
1.0078				110.064
1.0079				113.834
1.0080				117.669
1.0081				121.569
1.0082				125.534
1.0083				129.564
1.0084				133.659
1.0085				137.809
1.0086				142.019
1.0087				146.289
1.0088				150.619
1.0089				155.009
1.0090				159.459
1.0091				163.969
1.0092				168.539
1.0093				173.169
1.0094				177.859
1.0095				182.609
1.0096				187.419
1.0097				192.289
1.0098				197.219
1.0099				202.209
1.0100				207.259

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Table III : Solution for $K = 0.0349$

.100	.487	.004	.430	
.120	.301	.003	.430	
.140	.315	.005	.430	
.160	.330	.007	.431	
.180	.344	.009	.431	
.200	.358	.011	.431	
.220	.374	.017	.431	
.240	.387	.022	.431	
.260	.401	.029	.432	
.280	.415	.038	.432	
.300	.430	.049	.433	
.320	.444	.063	.433	
.340	.459	.080	.435	
.360	.473	.102	.436	
.380	.488	.128	.438	
.400	.503	.160	.441	
.420	.519	.199	.444	
.440	.533	.247	.448	
.460	.549	.306	.453	
.480	.565	.379	.460	
.500	.582	.468	.469	
.520	.600	.579	.480	
.540	.619	.717	.495	
.560	.637	.893	.513	
.580	.657	1.118	.537	
.600	.679	1.412	.569	
.620	.703	1.805	.610	
.640	.728	2.325	.665	
.660	.733	2.876	.679	
.680	.739	2.612	.693	
.700	.744	2.770	.708	
.720	.750	2.934	.724	
.740	.756	3.111	.742	
.760	.762	3.302	.760	
.780	.767	3.507	.777	
.800	.773	3.734	.800	
.820	.780	3.978	.823	
.840	.786	4.243	.847	
.860	.792	4.533	.873	
.880	.799	4.850	.902	
.900	.805	5.198	.932	
.920	.812	5.580	.965	
.940	.818	6.000	1.000	0.000
.960	.820	5.921	.920	.099
.980	.790	5.565	.792	.232
1.000	.764	5.093	.655	.361
1.004	.731	4.598	.527	.468
1.008	.692	4.130	.416	.547
1.012	.649	3.709	.323	.596
1.016	.603	3.342	.248	.619
1.020	.554	3.027	.187	.640
1.024	.503	2.758	.140	.663
1.028	.451	2.530	.102	.671
1.032	.399	2.336	.073	.688
1.036	.347	2.173	.051	.706
1.040	.297	2.034	.035	.720
1.044	.249	1.917	.023	.731
1.048	.203	1.819	.014	.741
1.052	.161	1.737	.009	.751
1.056	.123	1.668	.005	.761
1.060	.089	1.612	.002	.771
1.064	.060	1.567	.001	.781
1.068	.037	1.532	.000	.791
1.072	.019	1.507	.000	.801
1.076	.007	1.490	.000	.811
1.080	.001	1.482	.000	.821
1.084	.000	1.480	.000	.831
1.088	.000	1.480	.000	.841
1.092	.000	1.480	.000	.851
1.096	.000	1.480	.000	.861
1.100	.000	1.480	.000	.871

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Table IV : Solution for $\kappa = 0.349$

.400	.287	.004	1.165	
.480	.301	.005	1.165	
.460	.315	.007	1.165	
.460	.319	.010	1.165	
.480	.344	.011	1.165	
.500	.358	.010	1.165	
.520	.374	.026	1.166	
.540	.386	.035	1.166	
.560	.401	.046	1.166	
.580	.415	.060	1.167	
.600	.429	.077	1.168	
.620	.444	.099	1.170	
.640	.458	.125	1.171	
.660	.473	.158	1.174	
.680	.487	.199	1.177	
.700	.502	.248	1.181	
.720	.517	.308	1.186	
.740	.531	.381	1.192	
.760	.547	.469	1.200	
.780	.562	.576	1.211	
.800	.578	.705	1.224	
.820	.594	.861	1.240	
.840	.610	1.052	1.261	
.860	.627	1.285	1.287	
.880	.645	1.571	1.321	
.900	.664	1.928	1.362	
.920	.684	2.375	1.416	
.940	.705	2.946	1.484	
.960	.729	3.078	1.500	
.980	.743	3.218	1.517	
.992	.758	3.366	1.534	
.996	.772	3.521	1.553	
1.000	.787	3.636	1.572	
1.010	.798	3.860	1.593	
1.020	.806	4.044	1.615	
1.030	.812	4.239	1.638	
1.040	.816	4.446	1.662	
1.050	.819	4.666	1.687	
1.060	.821	4.900	1.715	
1.070	.822	5.149	1.747	
1.080	.823	5.415	1.774	
1.090	.824	5.698	1.806	
1.100	.825	6.000	1.840	0.000
1.110	.826	6.954	1.878	0.683
1.120	.827	7.302	1.904	1.258
1.130	.828	7.369	.907	1.679
1.140	.829	7.494	.618	1.967
1.150	.830	7.150	.475	2.115
1.160	.831	6.973	.365	2.232
1.170	.832	6.786	.279	2.324
1.180	.833	6.601	.211	2.491
1.190	.834	6.423	.157	2.690
1.200	.835	6.256	.115	1.950
1.210	.836	6.103	.083	1.782
1.220	.837	5.964	.058	1.593
1.230	.838	5.839	.040	1.353
1.240	.839	5.729	.026	1.188
1.250	.840	5.631	.016	.985
1.260	.841	5.547	.010	.791
1.270	.842	5.475	.005	.609
1.280	.843	5.415	.003	.444
1.290	.844	5.366	.001	.301
1.300	.845	5.328	.001	.182
1.310	.846	5.301	.000	.091
1.320	.847	5.283	.000	.030
1.330	.848	5.275	.000	.002
1.340	.849	5.275	.000	.000
1.350	.850	5.275	.000	.000